1. A random sample of size $n = 16$ has sample mean $\overline{x} = 82.1$ and sample variance $s^2 = 9.61$. The population is normal with mean and standard deviation μ and σ^2 respectively.
Find 90% confidence intervals for:

(a) μ, given that $\sigma = 3.0$;

(b) the 90-th percentile of the population, $\mu + z_{.9}\,\sigma$, given that $\sigma = 3$;

(c) μ, when σ is unknown;

(d) σ.

2. What is the minimum sample size you would recommend in order to estimate, with 95% confidence:

(a) the mean of a normal population to within 1.0 when (i) $\sigma = 3$; (ii) $\sigma = 30$;

(b) a population proportion, p, to within 0.02 when (i) no information about p can be assumed; (ii) it is known that $p \leq 0.15$.

3. How large a sample is required to estimate a standard deviation, σ, of a normal population in order to be 90% confident that σ is underestimated by at most 12%? (i.e. the estimator $\hat{\sigma}$ must satisfy $\sigma \leq 1.12\,\hat{\sigma}$ with probability 0.90.)

4. A random sample of 20 lifetimes of electronic components has mean 112.1 days. Find a 90% confidence interval for the mean lifetime μ:

(a) assuming the lifetimes to be normally distributed, and the standard deviation is: (i) known to be 13.2 days; (ii) estimated from the sample as 13.2 days;

(b) assuming the lifetimes have a gamma distribution with $\kappa = 3$ and unknown parameter θ.

Notes: If F is the cumulative distribution function for a χ^2–distribution with 120 degrees of freedom then $F(95.705) = 0.05$ and $F(146.567) = 0.95$.

5. The tensile strength, y, of a high–powered copper coil is believed to be normally distributed with mean μ and standard deviation σ. A sample of 14 high–powered copper coils yields the following tensile strength values (in units of kilogram per square centimetre):

429 187 247 305 218 366 253 246 379 296 227 198 280 259

The sample mean and standard deviation are $\overline{y} = 277.86$ and $s = 71.066$, respectively.

(a) Find a 90% confidence interval for the mean tensile strength assuming: (i) $\sigma = 65$ kg/cm^2; (ii) σ is unknown.

(b) Find a 90% confidence upper bound for σ.

(c) Taking the upper bound in (b) as an estimate of σ, what size sample would be required to estimate μ to within 20 kg/cm^2 with 90% confidence?

6. Arrivals of customers at a filling station are modelled as a homogeneous Poisson process with a rate of λ arrivals per hour, so that the number of arrivals in the interval $[0, t]$ has a Poisson distribution with mean $\mu = \lambda\, t$.

(a) Show that the time to the second arrival has a gam$(\theta, 2)$ distribution, where $\theta = 1/\lambda$.
Note: See notes for explanation of the parametrisation.

(b) The arrival rate is known to be $\lambda = 40$ arrivals per hour. Each arrival is recorded with probability 0.1, independently of previous arrivals or recordings. Find the probability that at most 2 arrivals are recorded in the first hour.

(c) If 20 successive inter-arrival times have a mean of 2 minutes, find a 90% confidence interval for the arrival rate λ.

This is the final problem sheet. Solutions to all questions should be handed up on Thursday 23rd November 2006